

①

ZA-332
Fluid Dynamics Research

224400

FILE COPY
Return to
ASTIA
ARLINGTON HALL STATION
ARLINGTON 12, VIRGINIA
ATTN: TISSE

ON THE MIXING PROBLEM OF AN AXI-SYMMETRIC FREE JET
INTO AIR INCLUDING CHEMICAL REACTIONS

Inge L. Ryhming

March 1961

PROJECT DEFENDER

This research was supported by Advanced
Projects Agency Order No. 116-60 (Project
Defender through Contract AF-19(604)-5554

ASTIA
JUN 21 1961

\$2.60

XEROX

AD No. —
ASTIA FILE COPY

257890



CONVAIR SAN DIEGO CONVAIR DIVISION GENERAL DYNAMICS CORPORATION



**BEST
AVAILABLE COPY**

**ON THE MIXING PROBLEM OF AN AXI-SYMMETRIC FREE JET
INTO AIR INCLUDING CHEMICAL REACTIONS**

Inge L. Ryning

March 1961

**This research was supported by Advanced
Projects Agency Order No. 116-60 (Project
Defender through Contract AF-19(604)-5554**

**CONVAIR
A Division of General Dynamics Corporation
San Diego**

ON THE MIXING PROBLEM OF AN AXI-SYMMETRIC FREE JET INTO AIR INCLUDING CHEMICAL REACTIONS

Inge L. Ryhming

ABSTRACT

Velocity and temperature fields together with specie concentrations are computed for an axially symmetric, supersonic, hot, free jet mixing with quiescent air. Based on simplifying assumptions regarding the chemistry involved, the problem can be divided in two parts. One of these parts is to determine the velocity and temperature field, ~~as was given previously by Pat.~~ The second part consists of integrating the continuity of the specie equations when the solution to the first part of the problem is obtained. Two different cases are considered frozen and equilibrium flow. For frozen flow it is shown that the specie concentrations are similar to the velocity distribution. For equilibrium flow the specie concentrations are obtained with a crude but simple analysis, in which the effects of diffusion and chemical reactions are combined.

INTRODUCTION

The gas dynamics of mixing phenomena in plane or axi-symmetric free jets into air has received considerable attention in the past.¹ The theory has been developed by using the usual boundary-layer approximations to determine laminar as well as turbulent mixing.

If the chemical phenomena which arise in mixing hot gases of various chemical composition are included in the analysis, the gas dynamics of the jet mixing problem becomes much more complicated. The difficulty arises mathematically from an increase in the number of dependent variables and also the development of new equations added to the problem, e.g., the continuity of specie equations are coupled with each other and with the energy equation of the reacting mixture.

At the present time very little is known about the rate constants in the production terms of the specie equations. So far, the two limiting cases (frozen flow or zero reaction rate, equilibrium flow or infinite reaction rate) or small deviations from them have in general been considered. Also, the chemistry involved has usually been confined to two component mixtures.²

In this investigation the velocity, temperature, and specie concentration distributions of an axi-symmetric and supersonic multi-component hot, free jet mixing with quiescent air is computed for frozen and equilibrium flow. The mixing process is supposed to occur under constant pressure, so that the pressure of the jet at the nozzle exit is completely balanced with the surrounding air pressure. In the analysis several rather crude assumptions are made to facilitate a simple solution. The simplifications imposed are mainly connected with the chemistry involved.

FLOW MODEL AND GOVERNING EQUATIONS

Consider a gas which is formed in the combustion chamber of a rocket motor and which then expands through the rocket nozzle. Depending on the mixture ratio of the rocket propellant (RP-1) and oxygen and on the expansion process through the nozzle, the gas will have different composition, pressure, temperature, and velocity distribution at the nozzle exit. Two different mixture ratios and two different expansion processes will be considered. First, it will be assumed the expansion follows a frozen state with a mixture ratio of 6 and second, the expansion is followed by an equilibrium state with a mixture ratio of 2.5. Furthermore, the expansion is supposed to be carried so far that the pressure at the nozzle exit

is balanced completely with the pressure of the surrounding still air at sea level or 1 atm. We also assume that the velocity, temperature, and concentration of the various species of the gas have a constant value over the nozzle exit.

The gas will be regarded as being composed of the following species: H_2O , CO_2 , CO , H_2 , O_2 , and a small amount of various other species which can be taken as non-reacting. As the temperature range we are considering is below $2600^\circ K$ we do not take into account any dissociation of O_2 and H_2 . The reactions which may take place are the formation of CO_2 and H_2 from CO and H_2O , and the formation of H_2O from H_2 and O_2 . These reactions are known to proceed according to a number of elementary steps. Here, however, the simplest possible form of chemical reactions will be assumed.

The air outside the jet is supposed to be composed of O_2 and N_2 ; the latter is regarded as an inert gas. The temperature of the air is assigned the value $T_a = 300^\circ K$.

A constant-pressure, laminar-mixing process will first be assumed, but later on in this investigation, turbulent mixing will also be considered. For calculating the properties of the described flow model, the equations of motion of the reacting mixture written with the usual boundary-layer approximations will suffice.^{1,3} Thereby, the effect of the temperature gradient on the speed of diffusion will be neglected. By introducing an axi-symmetric coordinate system with the axis of the jet being taken as the x-axis, the governing equations* are:

continuity of species:

$$\rho u \frac{\partial C_1}{\partial x} + \rho v \frac{\partial C_1}{\partial r} = \frac{1}{r} \frac{\partial}{\partial r} \left(r \rho D_1 \frac{\partial C_1}{\partial r} \right) + \dot{w}, \quad (1)$$

*For the exact equations of motion of a reacting mixture, see Hirschfelder, et al.⁴

over-all continuity:

$$\frac{\partial}{\partial x} (\rho u r) + \frac{\partial}{\partial r} (\rho u r) = 0, \quad (2)$$

momentum:

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial r} = \frac{1}{r} \frac{\partial}{\partial r} \left(r \mu \frac{\partial u}{\partial r} \right), \quad (3)$$

energy:

$$\rho u \frac{\partial h}{\partial x} + \rho v \frac{\partial h}{\partial r} = \frac{1}{r} \frac{\partial}{\partial r} \left[r \frac{\mu}{Pr} \left(\frac{\partial h}{\partial r} + \sum_1 (Le_1 - 1) H_1 \frac{\partial C_1}{\partial r} \right) \right] + \mu \left(\frac{\partial u}{\partial r} \right)^2 \quad (4)$$

Here ρ , u , v , and μ have their usual meaning, C_1 is the mass concentration of species i , D_1 the diffusion coefficient of species i , and \dot{W} is the production function of species i in the chemical reactions considered. In the energy equation, h stands for the enthalpy of the mixture

$$h = \sum_1 C_1 (h_1 - h_1^*) = \sum_1 C_1 H_1$$

where h_1^* is the heat evolved in the formation of component i at 0°K per unit mass, and Pr is the Prandtl number ($Pr = \frac{c_p \mu}{k}$), and Le_1 is the Lewis-Semenov number for species i ($Le_1 = \frac{c_p \rho D_1}{k}$).

The system of Equations (1) through (4) is, however, far too complicated for a simple solution to bring out the main features of the mixing problem. Further simplifications, therefore, are needed. One assumption which is generally employed and which will be adopted here is that the Lewis-Semenov number equals one. Then the energy equation is decoupled from the specie equations and a solution to Equations (2) through (4) can be obtained first. This simplification is justified because Le_1 has a value close to one in any actual case and because its variation for each specie is small in the flow field.

The equilibrium flow case was chosen because the chemical reactions which occur do not play a dominant role in the problem. Such a situation can be accomplished by choosing a low value for the mixture ratio of propellant to oxygen. This, in turn, implies that the effect of reactions on the specie concentration will, in general, be small, since the largest part of the reactants participating in the burning-type reactions already have reacted in the combustion chamber and given non-combustible products. Consequently, over the main part of the flow field, the \dot{W} term in Equation (1) will be small compared to the other terms involved except possibly for a thin layer in the mixing region where burning reactions can be stimulated by oxygen present in the surrounding air. Effectively, therefore, the equilibrium flow field will not differ significantly from a corresponding frozen flow field where no reactions occur and the \dot{W} term is zero.

Since the chemistry is of minor importance compared with the fluid mechanics of the problem, it seems reasonable to simplify the energy equation further. Hence, it is assumed that the enthalpy throughout can be approximated as

$$h = c_p T$$

where c_p is a mean value for the mixture. In this rather crude assumption h_1° is neglected entirely; i.e., the contribution to the enthalpy caused by the release of energy through reactions. However, for our equilibrium flow, the energy release will be small because of the chosen mixture ratio.

By applying these assumptions to the energy equation and the further assumptions that the density variation with temperature follows the ideal gas law (pressure constant)

$$\frac{\rho}{\rho_0} = \frac{T_0}{T} \quad (5)$$

and that the Prandtl number equals one, the problem is essentially divided into two parts. The first of these parts is to find the

velocity and temperature variation in the flow field. When these variations are determined the second part consists in finding the specie concentrations for the two different cases we are considering. The first part of this problem has been solved by Pai.⁵ However, the essential steps in the calculation are needed later on; therefore, they are repeated here for convenience.

THE VELOCITY AND TEMPERATURE FIELD

With the assumptions made in the last paragraph, the energy equation reads

$$\rho u c_p \frac{\partial T}{\partial x} + \rho v c_p \frac{\partial T}{\partial r} = \frac{1}{r} \frac{\partial}{\partial r} \left(k r \frac{\partial T}{\partial r} \right) + \mu \left(\frac{\partial u}{\partial r} \right)^2. \quad (6)$$

As shown by Crocco⁶ and others, a solution to Equation (6) is

$$T = A + B u - \frac{u^2}{2c_p}, \quad (7)$$

where A and B are constants determined by the initial conditions. The following non-dimensional quantities are now introduced in the over-all continuity and momentum equations:

$$\rho^* = \frac{\rho}{\rho_0}, \quad u^* = \frac{u}{u_0}, \quad v^* = \frac{v}{u_0}, \quad r^* = \frac{r}{r_0}, \quad x^* = \frac{x}{r_0}, \quad \mu^* = \frac{\mu}{\mu_0}, \quad T^* = \frac{T}{T_0},$$

where index 0 refers to the nozzle exit. Then the independent variables in the momentum equation are changed to x^* and ψ , where the stream function ψ is defined such that

$$\psi_{x^*} = -r^* \rho^* v^*, \quad \psi_{r^*} = r^* \rho^* u^*. \quad (9)$$

The result is

$$\frac{\partial u^*}{\partial x^*} = \frac{\mu_0}{r_0 \rho_0 u_0} \frac{\partial}{\partial \psi} \left(\mu^* \rho^* u^* r^{*2} \frac{\partial u^*}{\partial \psi} \right). \quad (10)$$

The over-all continuity equation is automatically satisfied by introducing the stream function Equation (9). In Equation (10) any temperature dependence of μ with T can be chosen. Here this temperature dependence is made as simple as possible by taking

$$\mu^* \rho^* = 1. \quad (11)$$

Furthermore, we introduce

$$\xi = x^* \frac{\mu_0}{r_0 \rho_0 u_0} \quad (12)$$

as a new independent variable. The momentum equation then is

$$\frac{\partial u^*}{\partial \xi} = \frac{\partial}{\partial \psi} \left(u^* r^{*2} \frac{\partial u^*}{\partial \psi} \right). \quad (13)$$

For the numerical integration of Equation (13) the behavior of the momentum equation on the x-axis is needed. This condition can be obtained from Equation (3) by taking the limit as r approaches zero and by using the symmetry conditions

$$v(x,0) = 0, \quad \frac{\partial}{\partial r} u(x,0) = 0.$$

The result expressed in the non-dimensional quantities is

$$\text{on } r^* = 0: \quad u^* \frac{\partial u^*}{\partial \xi} = 2T^{*2} \frac{\partial^2 u^*}{\partial r^{*2}}. \quad (14)$$

In view of the last paragraph the initial conditions for u^* and T^* are

$$\begin{aligned} u^* &= 1, \quad T^* = 1 \text{ for } r^* \leq 1 \\ u^* &= 0, \quad T^* = \frac{T_B}{T_0} \text{ for } r^* > 1. \end{aligned} \quad (15)$$

Equation (7) with Equation (15) gives now, in general,

$$T^* = \frac{T_a}{T_o} + \left(1 - \frac{T_a}{T_o}\right) u^* + \frac{u^* (1 - u^*)}{2 \frac{c_p T_o}{u_o^2}} \quad (16)$$

In Table 1 temperatures and concentrations of the various species at the nozzle exit are shown for the two cases investigated. From this table and Equation (16) the temperature variation in the flow field is given as

$$\begin{aligned} \text{Frozen flow} \quad T^* &= 0.1652 + 0.8348 u^* + \frac{u^* (1 - u^*)}{1.0517}, \\ \text{Equilibrium flow} \quad T^* &= 0.1161 + 0.8839 u^* + \frac{u^* (1 - u^*)}{1.2168}. \end{aligned} \quad (17)$$

Equation (13) can now be integrated numerically with the help of Equations (14), (15), and (17). In the numerical procedure it is further necessary to have the relation between the ψ and r^* . From Equation (9) this relation is

$$r^{*2} = 2 \int_0^{\psi} \frac{T^*}{u^*} d\psi.$$

The problem was programed for an IBM 704 computer and results from the computation are shown in Figures 1 and 2.

THE CALCULATION OF SPECIES IN THE FLOW FIELD

For calculating the various species in the flow field Equation (1) should be used. First we transform it to the x^* , ψ variables with the result

$$\frac{\partial C_1^*}{\partial x^*} = \frac{D_{1o}}{r_o u_o} \frac{\partial}{\partial \psi} \left(r^{*2} \rho^* u^* D_1^* \frac{\partial C_1^*}{\partial \psi} \right) + \frac{\dot{W} r_o}{\rho_o u_o C_{1o}} \quad (18)$$

TABLE 1. Initial Data for Frozen and Equilibrium Flow

Initial Data	Frozen Flow	Equilibrium Flow	Ambient Conditions
Mixture ratio	6	2.5	$P_a = 1 \text{ atm}$
T_o °K	1816	2585	$T_a = 300^\circ\text{K}$
u_o m/s	2223	2712	$\Gamma_{O_2} = 0.210$
Γ_{H_2O}	0.242	0.387	$\Gamma_{N_2} = 0.790$
Γ_{CO_2}	0.219	0.190	
Γ_{CO}	0.073	0.304	
Γ_{H_2}	0.010	0.097	
Γ_{O_2}	0.339	0.001	
$\Gamma_{\text{non react.}}$	0.117	0.021	
c_p K joule/kg	1.431	1.731	

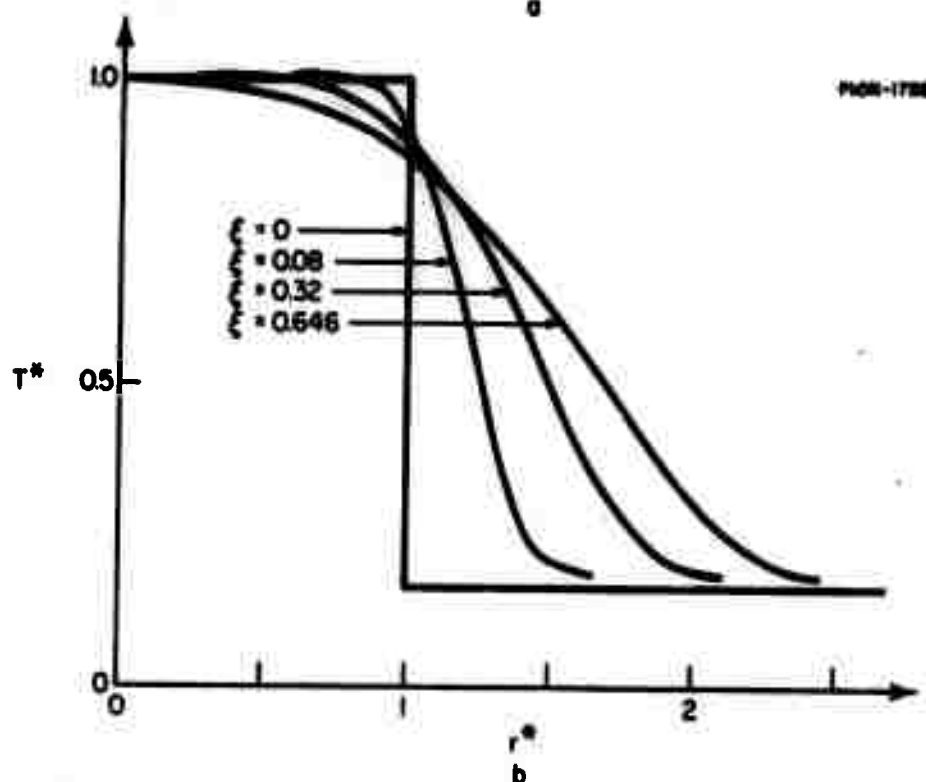
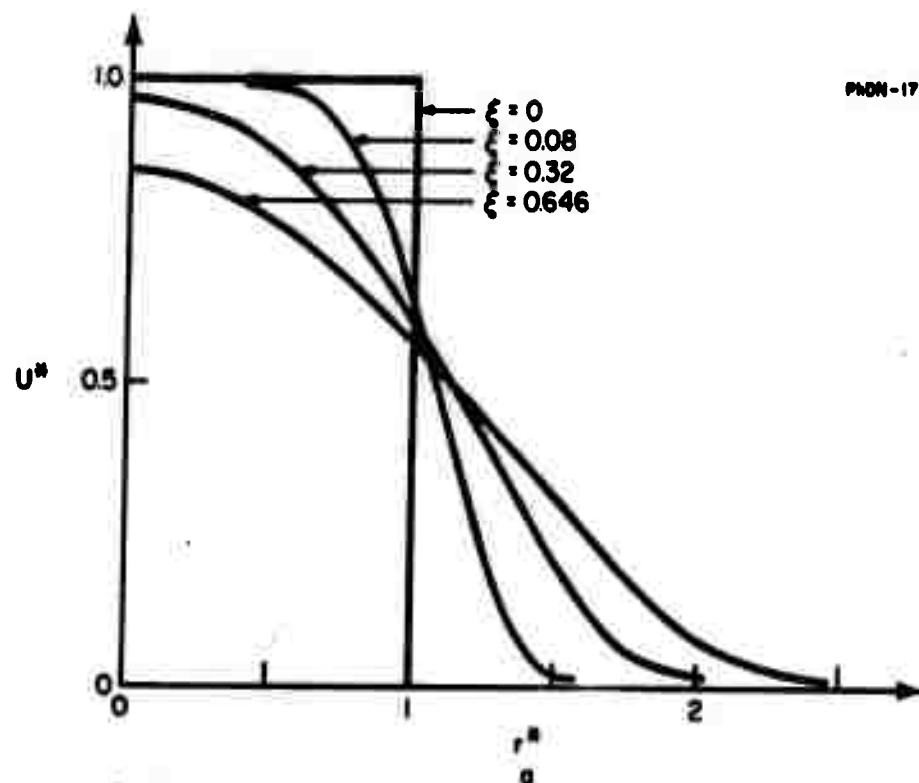


FIGURE 1 DISTRIBUTION OF VELOCITY U^* AND TEMPERATURE T^* IN THE FROZEN FLOW CASE

With the following approximate values $\mu_0 = 7 \cdot 10^{-5}$ kg/ms, $\rho_0 = 0.22$ kg/m³, $r_0 = 0.02$ m a value of $\Xi = 0.1$ is equivalent to: for $n = 0$: ($\xi = \Xi$) $x^* = 1.4 \cdot 10^4$; $n = 1$: $x^* = 1.7 \cdot 10^2$

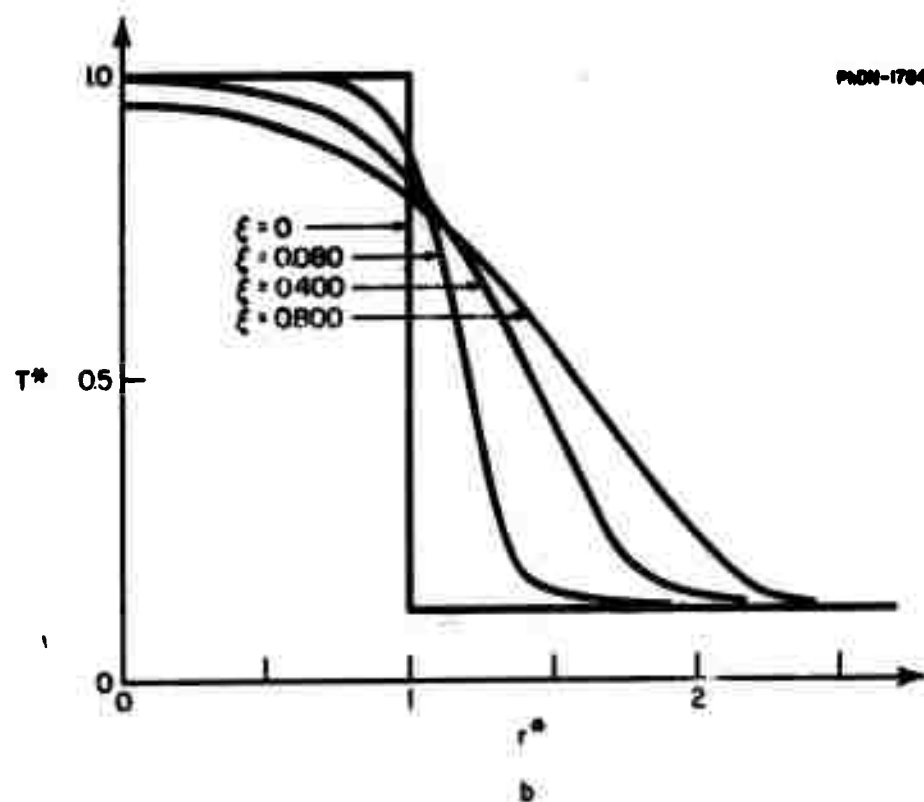
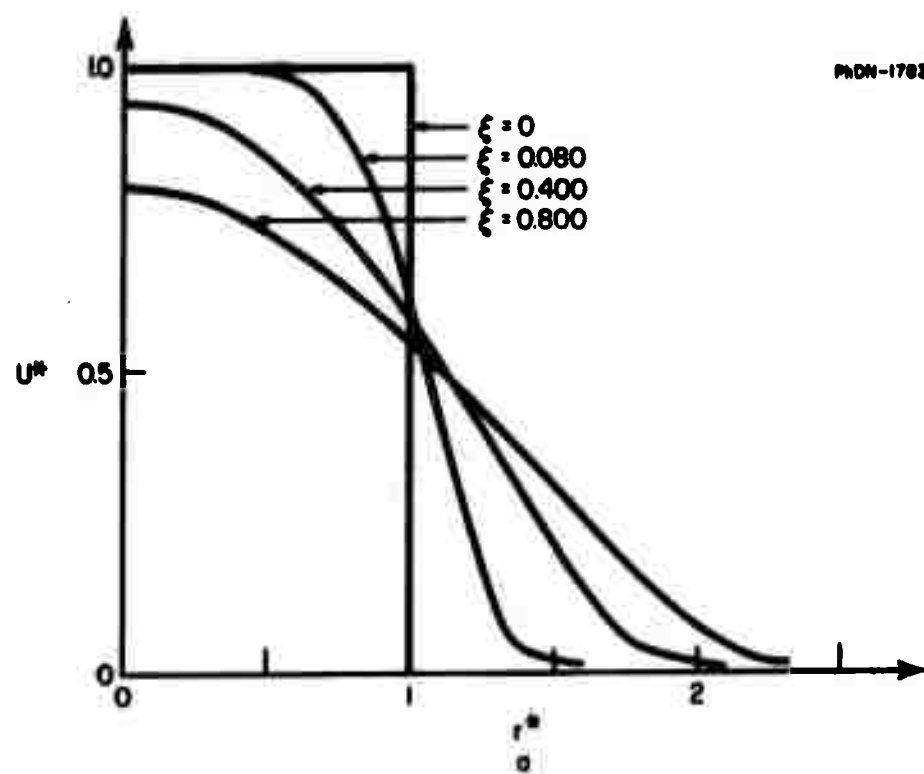


FIGURE 2 DISTRIBUTION OF VELOCITY U^* AND TEMPERATURE T^*
IN THE EQUILIBRIUM FLOW CASE

where $C_1^* = \frac{C_1}{C_{10}}$ and $D_1^* = \frac{D_1}{D_{10}}$.

But $Le_1 = \frac{c_p \rho D_1}{k} = 1$, $Pr = \frac{c_p \mu}{k} = 1$ and $c_p = \text{const.}$

Hence, $D_1^* \rho^* = \mu^*$ and since $\mu^* \rho^* = 1$ we have $\rho^{*2} D_1^* = 1$.

In terms of u_0 , ρ_0 and μ_0 we have

$$\frac{D_{10}}{r_0 u_0} = \frac{\mu_0}{r_0 \rho_0 u_0} \quad (19)$$

Thus, the same independent variables can be used as in the preceding paragraph, and Equation (18) can now be written as

$$\frac{\partial C_1^*}{\partial \xi} = \frac{\partial}{\partial \psi} \left(u^* r^{*2} \frac{\partial C_1^*}{\partial \psi} \right) + \dot{w} \frac{r_0^2}{\mu_0 C_{10}} \quad (20)$$

For frozen flow there are no reactions taking place and the production term in Equation (20) is zero. Equation (20) then has exactly the same form as Equation (13) since, as we are using non-dimensional quantities, it is immaterial if we speak about u^* or C_1^* . The distribution of a particular specie of the jet is similar to the u^* distribution, because for the jet species the initial conditions are

$$\begin{aligned} C_1^* &= 1 & \text{for } r^* \leq 1 \\ C_1^* &= 0 & \text{for } r^* > 1. \end{aligned} \quad (21)$$

We have $C_1^* = u^*$ or $C_1 = C_{10} u^*$.

For the O_2 and N_2 concentration of the ambient air the initial conditions are

$$\begin{aligned} C_1^* &= 1 & \text{for } r^* \geq 1; \\ C_1^* &= 0 & \text{for } r^* < 1. \end{aligned} \tag{22}$$

For these species, therefore, a new dependent variable

$$C_1^* = 1 - R$$

is chosen and we have then $R = u^* = 1 - C_1^*$ or $C_1 = C_{10} (1 - u^*)$.

In Figure 3 the specie concentrations are shown at two downstream stations of ξ .

Next, the specie concentrations for the equilibrium flow will be considered. It has already been pointed out that over the main part of the flow field the \dot{W} term is small compared with the other terms in the specie equations. This statement immediately suggests that the specie concentration for the equilibrium flow may be obtained with a perturbation of the pure diffusion distribution. A general way for doing this would be to first obtain C_1 with $\dot{W} = 0$ in Equation (20). A better approximation for C_1 can then be obtained by inserting the obtained C_1 's in the production term, which is essentially a product of the various C_1 's and make a new integration. This iterative procedure can be continued until the C_1 's obtained are close to those put in the \dot{W} term. However, this method is rather tedious and can only be performed with the help of a computer. It is far easier to use the concept of local chemical equilibrium with the pure diffusion distribution obtained from Equation (20) with $\dot{W} = 0$. The problem is then reduced to an algebraic one and a solution can be obtained based on the thermodynamic properties already computed in the first part of the problem. Let us consider this latter procedure.

The solution of Equation (20) with $\dot{W} = 0$ and the initial conditions Equation (21) and (22) applied to the initial equilibrium composition is first needed. Then at some point in the flow field we include a sample

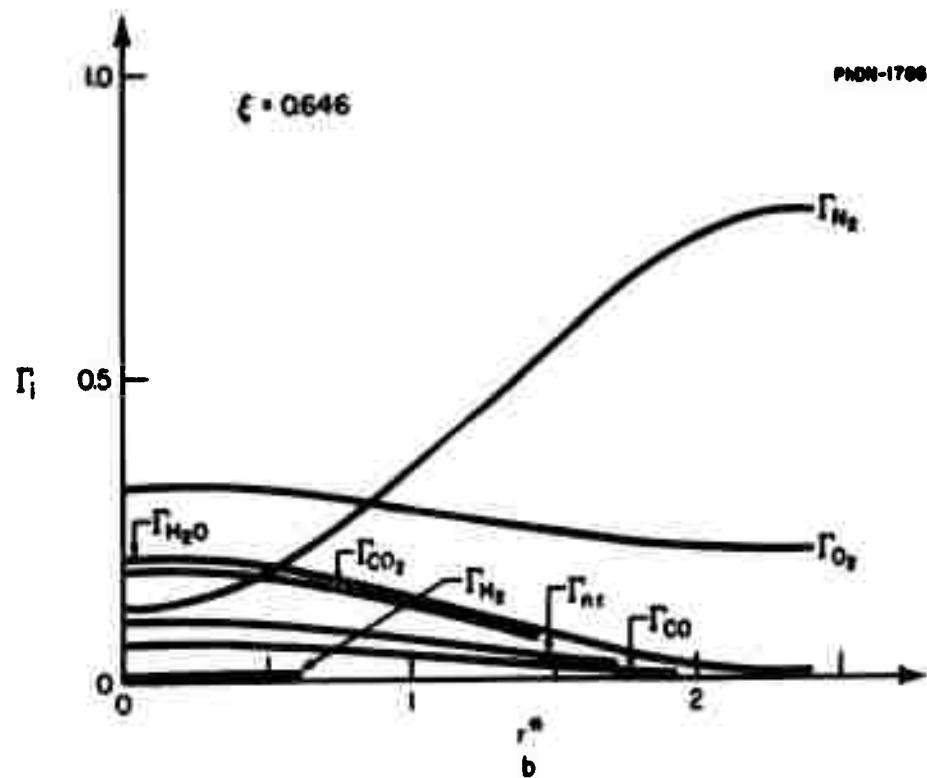
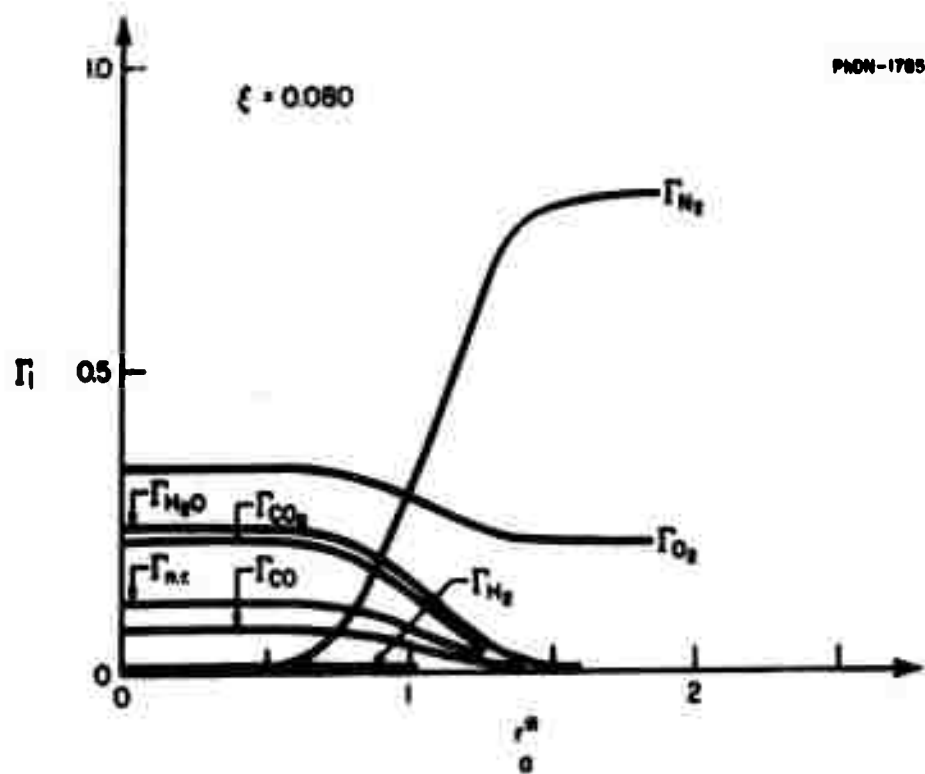
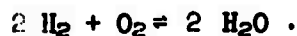


FIGURE 3 SPECIE CONCENTRATIONS AT $\xi = 0.080$ AND $\xi = 0.646$
FOR THE FROZEN FLOW

of gas of the now known diffusion composition in a small control volume and let this sample of gas come to chemical equilibrium at the prevailing temperature through the following reactions:



These reactions change the diffusion composition of the gas in the control volume. To compute the new composition we have, in accordance with the law of mass action,⁷ for the two reactions above ($p = 1 \text{ atm}$)

$$\ln K_{P_1} = \left(-\frac{F}{RT}\right)_{\text{CO}} + \left(-\frac{F}{RT}\right)_{\text{H}_2\text{O}} - \left(-\frac{F}{RT}\right)_{\text{CO}_2} - \left(-\frac{F}{RT}\right)_{\text{H}_2} = \ln \frac{\Gamma_{\text{CO}} \cdot \Gamma_{\text{H}_2\text{O}}}{\Gamma_{\text{CO}_2} \cdot \Gamma_{\text{H}_2}} ; \quad (23)$$

$$\ln K_{P_2} = 2\left(-\frac{F}{RT}\right)_{\text{H}_2\text{O}} - 2\left(-\frac{F}{RT}\right)_{\text{H}_2} - \left(-\frac{F}{RT}\right)_{\text{O}_2} = \ln \frac{\Gamma_{\text{H}_2\text{O}}^2}{\Gamma_{\text{H}_2}^2 \cdot \Gamma_{\text{O}_2}} . \quad (24)$$

Here $K_{P_{1,2}}$ are the reaction constants, F the free energy, and Γ_i the molecular concentration defined as

$$\Gamma_i = \frac{n_i}{\sum_i n_i}$$

where n_i is the number of gram-molecules of specie i . The relation between C_i and Γ_i is

$$C_i = \frac{\Gamma_i M_i}{\sum_i \Gamma_i M_i} ,$$

where M_i is the molecular weight of specie i . The value of $\left(-\frac{F}{RT}\right)_i$ can be obtained from standard gas tables.⁸

Three further equations for our five unknown Γ_i 's will be obtained from continuity relations. We have first

$$\sum_i \Gamma_i = 1 - \Gamma_{\text{non react}} - \Gamma_{N_2}. \quad (25)$$

Furthermore, in the small control volume the total amount of carbon to oxygen and hydrogen to oxygen does not change that is,

$$\frac{\sum C}{\sum O} = \text{const.}; \quad \frac{\sum H}{\sum O} = \text{const.} \quad (26)$$

which may in turn be translated to relations between the concentrations. The five Equations (23), (24), (25), and (26) constitute a non-linear system for the concentrations at each point which can be solved by numerical procedures.

In Figure 4, the equilibrium specie concentrations are shown at two downstream locations, however, the various curves are cut off at $r^* \approx 1$. It was found in the computation of the Γ_i 's that for a slightly larger value of r^* some of the Γ_i 's deviated more than 50% from their diffusion values. Therefore, the assumption that the \dot{W} term in Equation (20) is small in this region does not apply and the result of the simple calculation for these r^* values is not very satisfactory.

THE TURBULENT MIXING CASE

As has been shown by Pai,⁵ the momentum equations for the turbulent and the laminar mixing processes can be reduced to the same form, and we have the same solution for the same initial conditions. We shall briefly summarize this result and put it in a form suitable for the present investigation.

For turbulent jet mixing the momentum equation is^{1,5}:

$$\bar{\rho} \bar{u} \frac{\partial \bar{u}}{\partial x} + \bar{\rho} \bar{v} \frac{\partial \bar{u}}{\partial r} = \frac{\epsilon_1}{r} \left(\frac{x}{L} \right)^n \frac{\partial}{\partial r} \left(r \bar{\rho} \frac{\partial \bar{u}}{\partial r} \right) \quad (27)$$

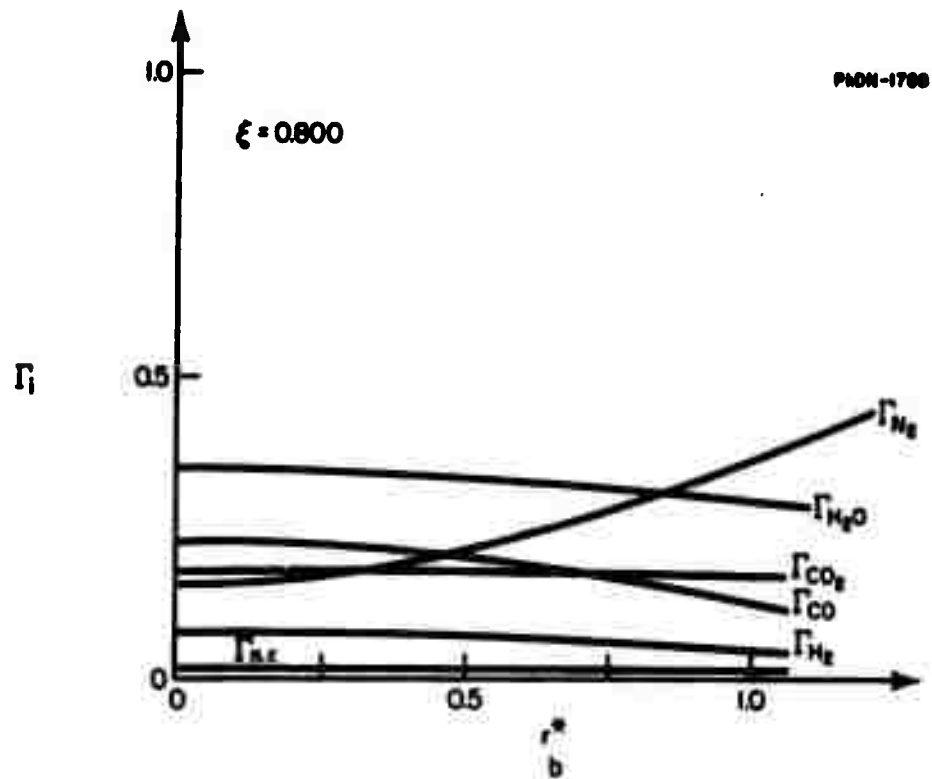
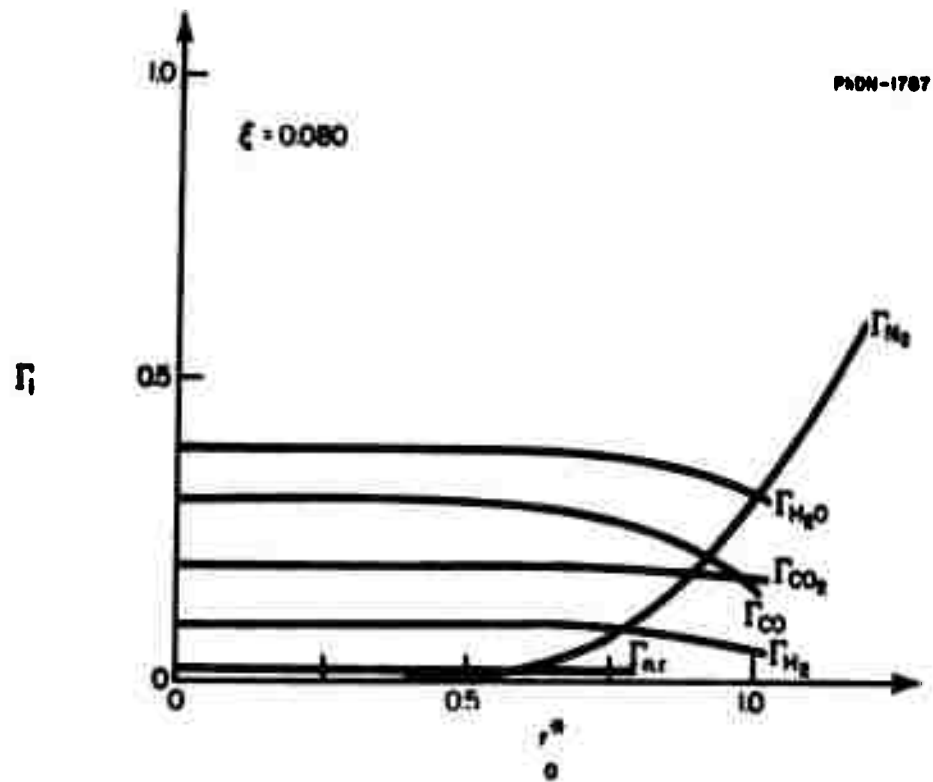


FIGURE 4 SPECIE CONCENTRATIONS AT $\xi=0.080$ AND $\xi=0.800$
FOR THE EQUILIBRIUM FLOW

where the bar denotes the average value of the variable. In general, the eddy kinematic viscosity coefficient ϵ may be written

$$\epsilon = \epsilon_1 \left(\frac{x}{L}\right)^n \quad (28)$$

where n lies between 0 and 1, ϵ_1 is an empirical constant and L is a reference length which in our case can be chosen as r_0 . Both ϵ_1 and n can be determined experimentally for the special case under consideration. In terms of non-dimensional quantities defined as

$$\tilde{\rho} = \frac{\bar{\rho}}{\rho_0}, \quad \tilde{u} = \frac{\bar{u}}{u_0}, \quad \tilde{v} = \frac{\bar{v}}{u_0}, \quad \tilde{\epsilon}_1 = \epsilon_1 \frac{\rho_0}{\mu_0} \quad (29)$$

and the turbulent stream function defined such that

$$\psi_{x^*} = - r^* \tilde{\rho} \tilde{v}, \quad \psi_{r^*} = r^* \tilde{\rho} \tilde{u} \quad (30)$$

the momentum equation is reduced to ($L = r_0$)

$$\frac{\partial \tilde{u}}{\partial x^*} = \frac{\mu_0}{\rho_0 u_0 r_0} x^{*n} \frac{\partial}{\partial \psi} \left(\tilde{\epsilon}_1 \tilde{\rho}^2 r^{*2} \tilde{u} \frac{\partial \tilde{u}}{\partial \psi} \right). \quad (31)$$

Here a new independent variable

$$\Xi = \frac{\mu_0}{\rho_0 u_0 r_0} \frac{x^{*n+1}}{n+1} \quad (32)$$

is chosen and by letting $\tilde{\epsilon}_1 \tilde{\rho}^2 = 1$ in analogy with Equation (11), Equation (31) takes exactly the same form as Equation (13). The resulting distribution for \tilde{u} can be taken from Figures 1 and 2 where only ξ has to be replaced by Ξ . The scale factor between the turbulent and laminar distribution is seen to be

$$\frac{\Xi}{\xi} = \frac{x^{*n}}{n+1}.$$

A similar argument can be shown to hold also for the specie concentrations.

DISCUSSION

The problem of a multi-component, hot, free jet mixing with cold air is very complex. To make the problem tractable several simplifying assumptions are necessary. With these assumptions in mind, however, it cannot be expected that the details of the mixing phenomenon will be very well represented. Especially for the equilibrium flow field calculation, for which the initial temperature is rather high, the conjectures regarding the enthalpy of the reacting mixture seems to be most severe. However, by choosing a low value of the mixture ratio the chemical effects can be made less important, so that the simplifications employed can be justified compared with others inferred in the problem.

Despite the crude model it is hoped that the main features of the mixing phenomena are brought out.

ACKNOWLEDGEMENTS

The author wishes to thank J. Bowyer, F. Harshbarger, A. Thomson, and H. Yoshihara for stimulating and constructive discussions. The programing and machine calculation was performed by F. Cochran and J. Doggett; their assistance is gratefully acknowledged. Thanks are also expressed to F. Boynton who furnished the data for Table 1.

SYMBOLS

x, r	axi-symmetric coordinates
ψ	stream function, defined in Equation (9) and (29)
ξ	variable defined in Equation (12)
Ξ	variable defined in Equation (31)
u, v	components of velocity vector
ρ	density
T	temperature °K
p	pressure
h	$\sum_i C_i (h_i - h_i^\circ) = \sum_i C_i H_i$, enthalpy of mixture
h_i°	heat evolved in formation of specie i at 0°K per unit mass
F	free energy
C_i	mass concentration of specie i
Γ_i	molecular concentration of specie i
D_i	diffusion coefficient of specie i
\dot{W}	mass rate of production of specie i per unit volume
μ	viscosity coefficient
c_p	specific heat at constant pressure
k	heat conduction coefficient
ϵ	eddy viscosity coefficient
Pr	$\frac{c_p \mu}{k}$ Prandtl number
Le_i	$\frac{c_p \rho D_i}{k}$ Lewis-Semenov number

A, B	constants defined in Equation (7)
K_p	reaction constant
ϵ_1, L, n	constants defined in Equation (27)
n_1	number of gram-molecules
M_1	molecular weight

Super- and Subscripts

\bar{p}, \bar{u}, \dots	average value
ρ^*, u^*, \dots	defined in Equation (8)
$\tilde{p}, \tilde{u}, \dots$	defined in Equation (28)
o	refers to nozzle exit
a	refers to ambient conditions

REFERENCES

1. Pai, S. T., "Fluid Dynamics of Jets," D. Van Nostrand Co., Inc., New York, 1954.
2. Kovitz, A. A. and Hoglund, R. F., "Laminar Parallel Stream Mixing with Dissociation and Recombination," The Physics of Fluids 3, 436-443, (1960).
3. Fay, J. A. and Riddell, F. R., "Theory of Stagnation Point Heat Transfer in Dissociated Air," J. Aero. Sci., Vol. 25, 73-85, (1958).
4. Hirschfelder, J. O., Curtiss, C. F., and Bird, R. B., "Molecular Theory of Gases and Liquids," John Wiley & Sons, Inc., New York, 1954.
5. Pai, S. T., "Axially Symmetrical Jet Mixing of a Compressible Fluid," Quart. Appl. Math., Vol. 10, 141-148, (1952).
6. Crocco, L., "Sulla Trasmissione del Calore da una Lamina Piana un Fluido Scorrente ad Alta Velocita," L'Aerotecnica, 12, 181-197, (1932).
7. Roberts, J. K. and Miller, A. R., "Heat and Thermodynamics," 5th Ed., Interscience Publishers, New York, 1960, 432-438.
8. Hilsenrath, J. et al., National Bureau of Standards, Circular Tables of Thermal Properties of Gases, Washington, 1955.

UNCLASSIFIED

UNCLASSIFIED